MEAM 5360: VISCOUS FLUID FLOW

Assignment 3: Creeping Flow around a Sphere

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Introduction

In this problem, we will be looking at the Creeping Flow around a sphere. To do this, we will use the stream-function formulation of the Stokes equation. This problem will show the useful characteristics of the stream-function and how the Stokes Flow Regime allows us to obtain a very simple governing equation; the Biharmonic function. Below, we have included a Sketch of the problem.

It is important to note that we will be using axisymmetric spherical coordinates. This tells us that all variables are only a function of r, θ and have no dependence on ϕ. Below, we have also included a list of assumptions made for this problem.

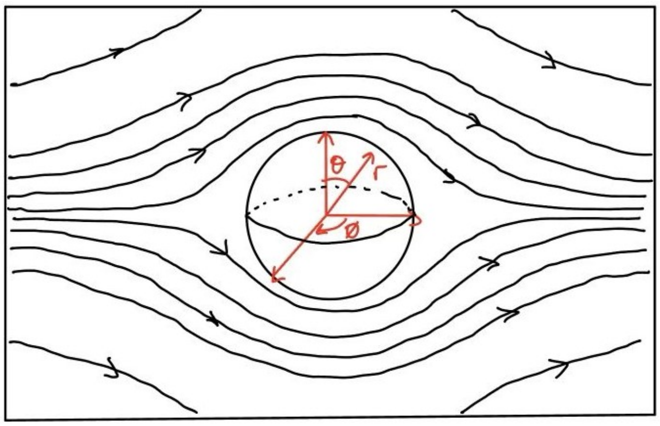


Figure 1: Sketch of Problem

Assumptions

1. Incompressible
2. Newtonian
3. Steady
4. $\frac{\partial}{\partial\theta} = 0$
5. $Re \ll 1$

The Biharmonic Function

In this section, we will derive our governing equation for this problem. To begin, we will invoke Assumption 5 to be able to start with the Stokes Equation below:

$$\nabla P = \mu\nabla^2\vec{v} \tag{1}$$

The first step is to take the curl of this function. Notice that pressure is a scalar-valued function and we know that $\nabla \times \nabla\phi = 0$ for any scalar function ϕ. In addition, since the Laplacian operator ∇^2 is a linear operator, we can swap the order of the curl operator and the Laplacian operator. This gives us:

$$0 = \mu\nabla^2(\nabla \times \vec{v}) = \mu\nabla^2\vec{\omega} \tag{2}$$

Here we have found the vorticity equation for stokes flow. This gives us a laplace equation for vorticity. It is known by Laplace's equation for the del operator that:

$$\nabla^2\vec{\omega} = \nabla(\nabla \cdot \vec{\omega}) - \nabla \times (\nabla \times \vec{\omega}) \tag{3}$$

The first term on the right-hand side includes the divergence of the vorticity. We know that this is the divergence of the curl of velocity, which we know is zero. That is to say:

$$\nabla \cdot \vec{\omega} = \nabla \cdot (\nabla \times \vec{u}) = 0 \tag{4}$$

From this, we can simplify Eq. (2) using Eq. (3) and what we have just derived to get to a new form of our governing equation:

$$\nabla \times (\nabla \times \vec{\omega}) = 0 \tag{5}$$

The next, we will note is the definition of the stream-function in spherical coordinates:

$$v_r = -\frac{1}{r^2 \sin(\theta)} \frac{\partial \psi}{\partial \theta}, v_\theta = \frac{1}{r \sin(\theta)} \frac{\partial \psi}{\partial r} \tag{6}$$

Since we are looking at a two-dimensional flow, we know that $\omega_r = \omega_\theta = 0$. This means that we only have to consider the out of page component of the vorticity. By the definition of the curl, we can say:

$$\omega_\phi = \frac{1}{r}\left(\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial \theta}\right) \tag{7}$$

Inputting the streamfunction relationships here, we get:

$$\omega_\phi = \frac{1}{r}\left(\frac{\partial}{\partial r}\left(r\frac{1}{r\sin(\theta)}\frac{\partial \psi}{\partial r}\right) - \frac{\partial}{\partial \theta}\left(-\frac{1}{r^2\sin(\theta)}\frac{\partial \psi}{\partial \theta}\right)\right)$$

$$\omega_\phi = \frac{1}{r}\left(\frac{1}{\sin(\theta)}\frac{\partial^2 \psi}{\partial r^2} + \left(\frac{1}{r^2\sin(\theta)}\frac{\partial^2 \psi}{\partial \theta^2} - \frac{\cot(\theta)}{r^2\sin(\theta)}\frac{\partial \psi}{\partial \theta}\right)\right)$$

$$\omega_\phi = \frac{1}{r\sin(\theta)}\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} - \frac{\cot(\theta)}{r^2}\frac{\partial}{\partial \theta}\right)\psi = \frac{E^2\psi}{r\sin(\theta)} \tag{8}$$

Next, we can plug this into our governing equation. Note that we can write:

$$\vec{\omega} = \omega_\phi \hat{\phi} \tag{9}$$

The first step to applying our governing equation is to take the curl of the vorticity. Doing this, we get:

$$\nabla \times \vec{\omega} = \frac{1}{r^2\sin(\theta)}\begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin(\theta)\hat{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ 0 & 0 & E^2\psi \end{vmatrix} \tag{10}$$

Taking the determinant to find the Curl, we get:

$$\nabla \times \vec{\omega} = \left(\frac{1}{r^2\sin(\theta)}\frac{\partial^(E^2\psi)}{\partial \theta}\right)\hat{r} - \left(\frac{1}{r\sin(\theta)}\frac{\partial(E^2\psi)}{\partial r}\right)\hat{\theta} \tag{11}$$

Taking the curl of this again by noticing that this is a 2D vector and we can once again only evaluate the ϕ component, we get:

$$\nabla \times (\nabla \times \vec{\omega}) = \frac{1}{r}\left(\frac{\partial}{\partial r}\left(r\frac{-1}{r\sin(\theta)}\frac{\partial(E^2\psi)}{\partial r}\right) - \frac{\partial}{\partial \theta}\left(\frac{1}{r^2\sin(\theta)}\frac{\partial(E^2\psi)}{\partial \theta}\right)\right)$$

$$\nabla \times (\nabla \times \vec{\omega}) = -\frac{1}{r}\left(\frac{1}{\sin(\theta)}\frac{\partial^2(E^2\psi)}{\partial r^2} + \left(\frac{1}{r^2\sin(\theta)}\frac{\partial^2(E^2\psi)}{\partial \theta^2} - \frac{\cot(\theta)}{r^2\sin(\theta)}\frac{\partial(E^2\psi)}{\partial \theta}\right)\right)$$

$$\nabla \times (\nabla \times \vec{\omega}) = -\frac{1}{r\sin(\theta)}\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} - \frac{\cot(\theta)}{r^2}\frac{\partial}{\partial \theta}\right)E^2\psi = -\frac{E^2(E^2\psi)}{r\sin(\theta)} \tag{12}$$

Substituting this into our governing equation, and multiplying both sides by $-1/(r\sin(\theta)$

$$E^2(E^2\psi) = E^4\psi = 0 \tag{13}$$

This is the result we were looking to prove, so now we are ready to focus more on the physics.

Boundary Conditions - Discovery

Since this is a fourth order problem, we expect four boundary conditions. However, since we only care about the velocity components, which are all derivatives of the streamfunction, we need one less boundary condition. This means we need a total of 3 boundary conditions for the stream-function. We are given two in the form of the no slip boundary condition at the boundary. Since we know $V_r = 0$ and $V_\theta = 0$ at the surface, we know that both first derivatives of the streamfunction at $r = R$ has to be zero. That is to say:

$$\left.\frac{\partial \psi}{\partial r}\right|_{r=R} = 0 \tag{14}$$

$$\left.\frac{\partial \psi}{\partial \theta}\right|_{r=R} = 0 \tag{15}$$

The final boundary states that far from the sphere, we return to the freestream velocity. If we divide the freestream velocity into components, we get the following relationship:

$$-\frac{1}{r^2 \sin(\theta)}\frac{\partial \psi}{\partial \theta} = V_\infty \cos(\theta) \tag{16}$$

$$\frac{1}{r\sin(\theta)}\frac{\partial \psi}{\partial r} = -V_\infty \sin(\theta) \tag{17}$$

This gives us the system of ODE's:

$$\frac{\partial \psi}{\partial \theta} = -V_\infty r^2 \sin(\theta)\cos(\theta) \tag{18}$$

$$\frac{\partial \psi}{\partial r} = -V_\infty r \sin^2(\theta) \tag{19}$$

It is clear to see from here that integration of both ODE's gives the same result, barring a constant. However, since this constant is not a function of r or θ and we only care about the derivatives of ψ in the future, we can call this constant whatever we want. That gives us the final solution:

$$\psi = -\frac{1}{2}V_\infty r^2 \sin^2(\theta) \tag{20}$$

This completes the necessary boundary conditions.

Separation of Variables

We established that over governing equation was

$$E^2(E^2\psi) = 0$$

We now want to assume that, based on the boundary conditions, we have a streamfunction of the form:

$$\psi = f(r)\sin^2(\theta) \tag{21}$$

To see what this does, let us find $E^2\psi$ Assuming this is the case. Doing this, can say:

$$\begin{aligned} E^2\psi &= \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} - \frac{\cot(\theta)}{r^2}\frac{\partial}{\partial \theta}\right) f\sin^2(\theta) \\ &= \left(\sin^2(\theta)f'' - \frac{2f}{r^2}(\sin^2(\theta) - \cos^2(\theta)) - \frac{2f\cos^2(\theta)}{r^2}\right) \\ &= \sin^2(\theta)\left(\frac{\partial^2}{\partial r^2} - \frac{2}{r^2}\right) f = w\sin^2(\theta) \end{aligned}$$

Where $w = \left(\frac{\partial^2}{\partial r^2} - \frac{2}{r^2}\right) f$. Notice that the input and the output have the same form, where w has replaced f. Given this, we can say:

$$E^2(E^2\psi) = E^2(w\sin^2(\theta)) \tag{22}$$
$$= \sin^2(\theta)\left(\frac{\partial^2}{\partial r^2} - \frac{2}{r^2}\right) w \tag{23}$$
$$= \sin^2(\theta)\left(\frac{\partial^2}{\partial r^2} - \frac{2}{r^2}\right)\left(\frac{\partial^2}{\partial r^2} - \frac{2}{r^2}\right) f \tag{24}$$

Given that this will be set equal to zero, we can remove the $\sin^2(\theta)$. This bring our governing equation for the r component to:

$$\left(\frac{\partial^2}{\partial r^2} - \frac{2}{r^2}\right)\left(\frac{\partial^2}{\partial r^2} - \frac{2}{r^2}\right) f = 0 \tag{25}$$

We are told this is an Euler Differential equation. This has solutions of the form $f = Cr^n$. Plugging this in, we get:

$$\left(\frac{\partial^2}{\partial r^2} - \frac{2}{r^2}\right)\left(\frac{\partial^2}{\partial r^2} - \frac{2}{r^2}\right) Cr^n = 0$$
$$\left(\frac{\partial^2}{\partial r^2} - \frac{2}{r^2}\right)\left(n(n-1)Cr^{n-2} - 2Cr^{n-2}\right) = 0$$
$$n(n-1)(n-2)(n-3)Cr^{n-4} - 2n(n-1)Cr^{n-4} - 2(n-2)(n-3)Cr^{n-4} + 4Cr^{n-4} = 0$$
$$n(n-1)(n-2)(n-3) - 2n(n-1) - 2(n-2)(n-3) + 4 = 0$$
$$n^4 - 6n^3 + 11x^2 - 6x - 2n^2 + 2n - 2n^2 + 10n - 12 + 4 = 0$$
$$n^4 - 6n^3 + 7n^2 + 6n - 8 = 0$$

To solve this final quartic equation, one can take many approaches. We chose to solve this using MATLAB, getting the possible values of n as $n = -1, 1, 2, 4$. This matches out expectations. This tells us:

$$f(r) = \frac{A}{r} + Br + Cr^2 + Dr^4 \tag{26}$$

Boundary Conditions - Application

Putting the above result together with the initial prediction, we get:

$$\psi = \left(\frac{A}{r} + Br + Cr^2 + Dr^4\right)\sin^2(\theta) \tag{27}$$

Now that we have our general solution, we need to start applying our boundary conditions. If we take the limit as $r \to \infty$, we would expect the highest order term to be the only remaining term and we also expect this term to be r^2. Since our general solution has an r^4 term which we know cannot exist, we can say $D = 0$. Now, we can apply our third boundary condition to get:

$$Cr^2\sin^2(\theta) = -\frac{1}{2}V_\infty r^2\sin^2(\theta) \tag{28}$$

This clearly shows us that $C = V_\infty/2$. Plugging in the above values for C, D, we get:

$$\psi = \left(\frac{A}{r} + Br - \frac{V_\infty}{2}r^2\right)\sin^2(\theta) \tag{29}$$

If we now convert our stream-function back into velocities by taking the appropriate definitions, we get:

$$v_r = -\frac{1}{r^2\sin(\theta)}\frac{\partial\psi}{\partial\theta}$$
$$= -\frac{1}{r^2\sin(\theta)}\left(\frac{A}{r} + Br - \frac{V_\infty}{2}r^2\right)\cdot 2\sin(\theta)\cos(\theta)$$

$$v_r = \left(V_\infty - \frac{2A}{r^3} - \frac{2B}{r}\right)\cos(\theta) \tag{30}$$

Doing the same process for v_θ we get:

$$v_\theta = \frac{1}{r\sin(\theta)}\frac{\partial \psi}{\partial r}$$

$$= \frac{1}{r\sin(\theta)}\left(-\frac{A}{r^2} + B - V_\infty r\right)\sin^2(\theta)$$

$$v_\theta = \left(-V_\infty - \frac{A}{r^3} + \frac{B}{r}\right)\sin(\theta) \tag{31}$$

If we plug in $r = R$ for both equations, apply the no-slip condition, and divide the $\sin(\theta)$ and $\cos(\theta)$ out of both equations, we get:

$$V_\infty - \frac{2A}{R^3} - \frac{2B}{R} = 0 \tag{32}$$

$$-V_\infty - \frac{A}{R^3} + \frac{B}{R} = 0 \tag{33}$$

This is a system of two equations for two unknowns. If we subtract two of the second equation from the first equation we get:

$$3V_\infty - \frac{4B}{R} = 0$$

$$B = \frac{3}{4}V_\infty R \tag{34}$$

Plugging this back into the second equation, we can find:

$$-V_\infty - \frac{A}{R^3} + \frac{3}{4}\frac{V_\infty R}{R} = 0$$

$$A = -\frac{1}{4}V_\infty R^3 \tag{35}$$

Plugging both of these equations into the velocity equations and noticing that all terms have a V_∞ we can divide out, we get:

$$\frac{v_r}{V_\infty} = \left[1 + \frac{1}{2}\left(\frac{R}{r}\right)^3 - \frac{3}{2}\left(\frac{R}{r}\right)\right]\cos(\theta) \tag{36}$$

$$\frac{v_\theta}{V_\infty} = -\left[1 - \frac{1}{4}\left(\frac{R}{r}\right)^3 - \frac{3}{4}\left(\frac{R}{r}\right)\right]\sin(\theta) \tag{37}$$

This matches the solution we expected. Below is a vector field of the velocity profile along with streamlines.

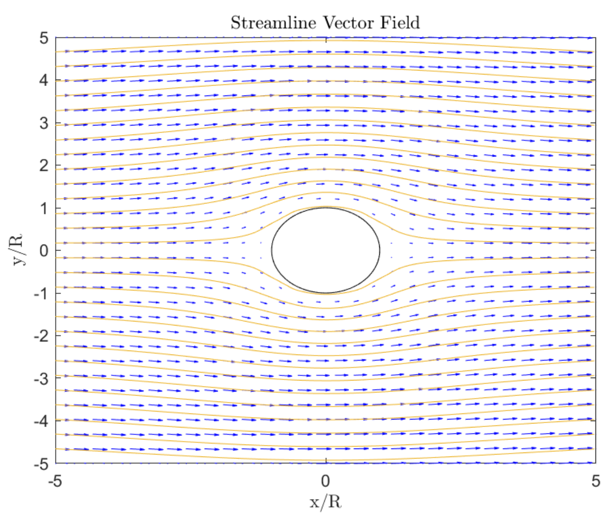


Figure 2: Vector Field and Streamlines

Pressure and Shear Stress

To begin, let us look at the pressure distribution. Recall that we can use the Laplace formula to say:

$$\nabla P = -\nabla \times \nabla \vec{v}\mu \tag{38}$$

If we expand the cross-products, we can say:

$$\nabla \times \vec{v} = \frac{1}{r}\left(\frac{\partial}{\partial r}(rv_\theta) - \frac{\partial v_r}{\partial \theta}\right)$$

Plugging in and expanding, we get:

$$\frac{\partial}{\partial r}(rv_\theta) = -V_\infty \left[1 + \frac{1}{2}\left(\frac{R}{r}\right)^3\right]\sin(\theta)$$

$$\frac{\partial v_r}{\partial \theta} = -V_\infty \left[1 - \frac{3}{2}\left(\frac{R}{r}\right) + \frac{1}{2}\left(\frac{R}{r}\right)^3\right]\sin(\theta)$$

Given this, we can say:

$$\nabla \times \vec{v} = -\frac{3RV_\infty}{2r^2}\sin(\theta)\hat{\phi} \tag{39}$$

Since want to find the negative value of the curl of the curl of velocity, we can remove the negative from this equation to both get exactly what we want and to simplify the equations slightly. Now, we need to take the curl of this. However, we specifically need the r component of the curl. We know that this is given by:

$$\left(\nabla \times \vec{A}\right)_r = \frac{1}{r\sin(\theta)}\left(\frac{\partial}{\partial \theta}(A_\phi \sin\theta) - \frac{\partial A_\theta}{\partial \phi}\right) \tag{40}$$

We know in our case that $A_\theta = 0$ so this becomes:

$$\begin{aligned}(-\nabla \times \nabla \times \vec{v})_r &= \frac{1}{r\sin(\theta)}\frac{\partial}{\partial \theta}\left(\frac{3RV_\infty}{2r^2}\sin^2(\theta)\right)\\ &= \frac{1}{r\sin(\theta)}\frac{3RV_\infty}{r^2}\sin(\theta)\cos(\theta)\\ &= \frac{3RV_\infty}{r^3}\cos(\theta)\end{aligned}$$

By our momentum balance, we get:

$$\frac{\partial P}{\partial r} = \frac{3\mu RV_\infty}{r^3}\cos(\theta) \tag{41}$$

Integrating this from $r = r$ to $r = \infty$ and assuming a freestream pressure P_∞, we can say this becomes:

$$P_\infty - P = -\frac{3}{2}\frac{\mu RV_\infty}{r^2}\cos(\theta)\Big|_r^\infty \tag{42}$$

It is clear that the upper bound of the right side goes to zero. This gives us:

$$P_\infty - P = \frac{3}{2}\frac{\mu V_\infty}{R}\left(\frac{R}{r}\right)^2\cos(\theta)$$

$$P = P_\infty - \frac{3}{2}\frac{\mu V_\infty}{R}\left(\frac{R}{r}\right)^2\cos(\theta) \tag{43}$$

Notice here that we are using the Piezometric pressures. If we converge off the this convention and include the gravity terms, calling $z = 0$ at $r = 0$, we get:

$$P = P_\infty - \rho g z - \frac{3}{2}\frac{\mu V_\infty}{R}\left(\frac{R}{r}\right)^2\cos(\theta) \tag{44}$$

The last thing to do here is to calculate the Viscous Stresses. To do this, recall that we are solving a two dimension problem. This means that $\tau_{r\phi} = \tau_{r\phi} = \tau_{\phi\phi} = 0$ Due to symmetry, this leaves us with only 3 components of stress to solve for. In spherical coordinates, these components can be written as:

$$\tau_{rr} = \mu \left[2\frac{\partial v_r}{\partial r} - \frac{2}{3}\nabla \cdot \vec{v} \right] \tag{45}$$

$$\tau_{\theta\theta} = \mu \left[2\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r} \right) - \frac{2}{3}\nabla \cdot \vec{v} \right] \tag{46}$$

$$\tau_{r\theta} = \mu \left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r} \right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta} \right] \tag{47}$$

Since we have an incompressible fluid, we know $\nabla \cdot \vec{v} = 0$. Algebra can show us that:

$$\frac{\partial v_r}{\partial r} = \frac{3}{2}V_\infty \left[\left(\frac{R}{r^2} \right) - \left(\frac{R^3}{r^4} \right) \right] \cos(\theta)$$

$$\frac{\partial v_r}{\partial \theta} = -V_\infty \left[1 - \frac{3}{2}\left(\frac{R}{r} + \frac{1}{2}\left(\frac{R}{r} \right)^3 \right) \right] \sin(\theta)$$

$$\frac{\partial v_\theta}{\partial \theta} = -V_\infty \left[1 - \frac{3}{4}\left(\frac{R}{r} \right) - \frac{1}{4}\left(\frac{R}{r} \right)^3 \right] \cos(\theta)$$

$$\frac{\partial}{\partial r}\left(\frac{v_\theta}{r} \right) = -V_\infty \left[-\frac{1}{r^2} + \frac{3}{2}\left(\frac{R}{r^3} \right) + \left(\frac{R^3}{r^5} \right) \right]$$

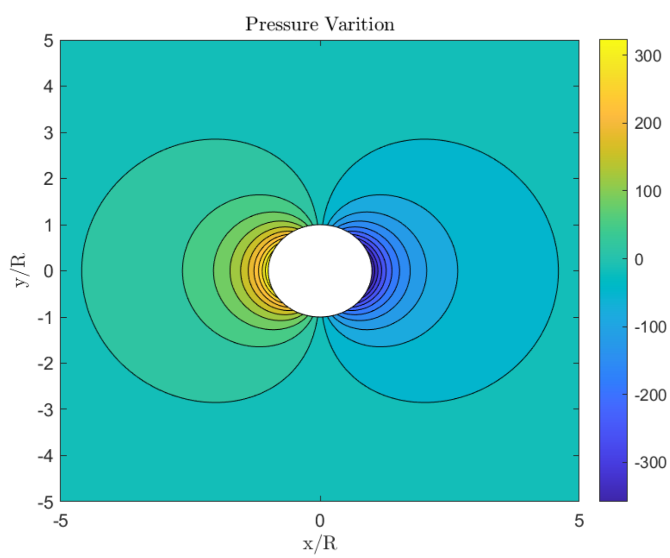
Given these, and the velocity components, we can evaluate the stresses using algebra. Doing this, we get:

$$\tau_{rr} = \frac{3\mu V_\infty}{R} \left[\left(\frac{R}{r} \right)^2 - \left(\frac{R}{r} \right)^4 \right] \cos(\theta) \tag{48}$$

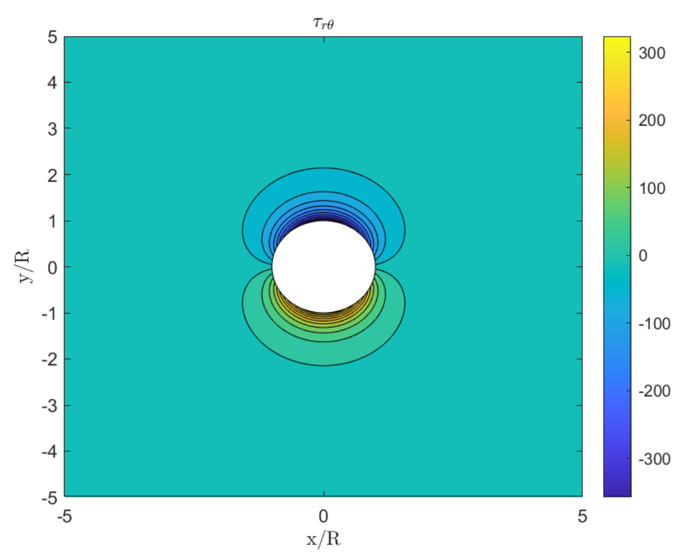
$$\tau_{\theta\theta} = \frac{3\mu V_\infty}{2R} \left[\left(\frac{R}{r} \right)^2 - \left(\frac{R}{r} \right)^4 \right] \cos(\theta) \tag{49}$$

$$\tau_{r\theta} = -\frac{3\mu V_\infty}{2R} \left(\frac{R}{r} \right)^4 \sin(\theta) \tag{50}$$

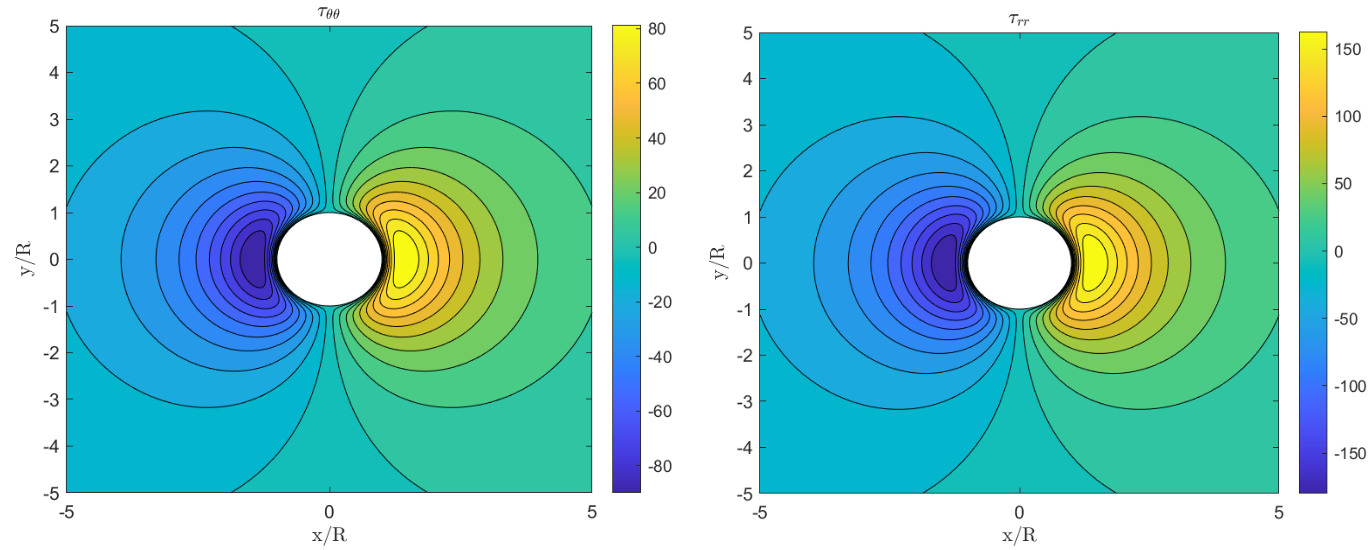
If we graph each of these results, we get:



A



B



C **D**

Figure 3: A) Pressure Contour B) $\tau_{r\theta}$ Contour C) $\tau_{\theta\theta}$ Contour D) τ_{rr}

As can be seen in these graphs, the pressure is high in front and low in the back. However, there is a symmetry to the pressure. All the graphs have a symmetry from left to right. That is also the flow direction. This is characteristic of Stokes Flow, as it does not have turbulent mixing.

The $\tau_{r\theta}$ graph also has interesting behavior, being symmetric along the y axis but negatively symmetric along the x axis. This is because once we take the dot product with the surface normal, we need the viscous forces to act in the positive x direction. However, since the normal direction changes direction the stress must change sign to maintain the net force direction.

Finally, we can see that the Main diagonal components of the viscous stress have identical graphs. The only difference is tha the $\theta\theta$ component is half the magnitude of the rr component. This is expected from the equations, and reflected in the graphs also.

Appendix

0.1 MATLAB Code

```
clear;
clc;
close all;
set(0,'defaultTextInterpreter','latex')

%% Define Parameters
mu = 1.5e-5;
Vinfty = 1;
rho = 1.2;
Pinfty = 100000;

Re = 0.01;
D = Re*mu/(rho*Vinfty);
R = D/2;

xset = linspace(-5*R,5*R,1000);
yset = xset;

[xset,yset] = meshgrid(xset,yset);
[thetaset,rset] = cart2pol(xset,yset);

% Find Pressure and Stresses
P = Pressure(rset,thetaset,Pinfty,Vinfty,mu,R);
[TauRR,TauTT,TauRT] = Stresses(rset,thetaset,Vinfty,mu,R);
% Remove Points Inside Circle
for i = 1:length(xset)
    for j = 1:length(yset)
        d = sqrt(xset(i,j)^2+yset(i,j)^2);
        if(d <= R)
            P(i,j) = NaN;
            TauRR(i,j) = NaN;
            TauTT(i,j) = NaN;
            TauRT(i,j) = NaN;
        end
    end
end
tset = linspace(0,2*pi,100);

figure;
contourf(xset/R,yset/R,P-Pinfty,20);
hold on;
plot(real(exp(1i*tset)),imag(exp(1i*tset)),'k-');
colorbar;
xlabel("x/R");
ylabel("y/R");
title("Pressure Varition");
xlim([-5 5]);
ylim([-5 5]);
exportgraphics(gcf,"Pressure.png");

figure;
contourf(xset/R,yset/R,TauRR,20);
```

```
hold on;
plot(real(exp(1i*tset)),imag(exp(1i*tset)),'k-');
colorbar;
xlabel("x/R");
ylabel("y/R");
title("$\tau_{rr}$")
xlim([-5 5]);
ylim([-5 5]);
exportgraphics(gcf,"tauRR.png");

figure;
contourf(xset/R,yset/R,TauTT,20);
hold on;
plot(real(exp(1i*tset)),imag(exp(1i*tset)),'k-');
colorbar;
xlabel("x/R");
ylabel("y/R");
title("$\tau_{\theta\theta}$")
xlim([-5 5]);
ylim([-5 5]);
exportgraphics(gcf,"tauTT.png");

figure;
contourf(xset/R,yset/R,TauRT,20);
hold on;
plot(real(exp(1i*tset)),imag(exp(1i*tset)),'k-');
colorbar;
xlabel("x/R");
ylabel("y/R");
title("$\tau_{r\theta}$")
xlim([-5 5]);
ylim([-5 5]);
exportgraphics(gcf,"tauRT.png");
%% Velocity
xset = linspace(-5*R,5*R,30);
yset = xset;

[xset,yset] = meshgrid(xset,yset);
[thetaset,rset] = cart2pol(xset,yset);

[vr,vtheta] = Velocity(rset,thetaset,R,Vinfty);

vx = vr.*cos(thetaset)-vtheta.*sin(thetaset);
vy = vr.*sin(thetaset)+vtheta.*cos(thetaset);

for i = 1:length(xset)
    for j = 1:length(yset)
        d = sqrt(xset(i,j)^2+yset(i,j)^2);
        if(d <= R)
            vx(i,j) = 0;
            vy(i,j) = 0;
        end
    end
end

figure;
```

```
quiver(xset/R,yset/R,vx,vy,0.6,'b');
hold on;
plot(real(exp(1i*tset)),imag(exp(1i*tset)),'k-');
lines = streamline(xset/R,yset/R,vx,vy,xset(:,1)/R,yset(:,1)/R);
xlim([-5 5]);
ylim([-5 5]);
xlabel("x/R");
ylabel("y/R");
title("Streamline Vector Field")
exportgraphics(gcf,"StreamlineVector.png");
%% Helper Functions
function [vr,vtheta] = Velocity(r,theta,R,Vinfty)
    normR = R./r;
    termR = 1+0.5*normR.^3-1.5*normR;
    termTheta = -(1-0.25*normR.^3-0.75*normR);
    vr = Vinfty*termR.*cos(theta);
    vtheta = Vinfty*termTheta.*sin(theta);
end

function P = Pressure(r,theta,Pinfty,Vinfty,mu,R)
    normR = R./r;
    const = 1.5*Vinfty*mu/R;
    P = Pinfty - const*normR.^2.*cos(theta);
end

function [TauRR,TauThetaTheta,TauRTheta] = Stresses(r,theta,Vinfty,mu,R)
    normR = R./r;
    % RR
    const = 3*mu*Vinfty/R;
    termR = normR.^2-normR.^4;
    TauRR = const*termR.*cos(theta);

    %Theta Theta
    TauThetaTheta = TauRR/2;

    % R Theta
    const = const/2;
    TauRTheta = const*normR.^4.*sin(theta);
end
```